Greedy Algorithms

Greedy Algorithm

- Most straight forward algorithm
- They are easy to invent, easy to implement and *– when they work –* efficient.
- Typically used to solve **Optimization Problems**
- *Crude Approach*, so many problems cannot be solved correctly.

Making Change (1) Problem

- Suppose, a country has following coins:

  100 paisa, 25 paisa, 10 paisa, 5 paisa & 1 paisa

- **Our Problem** is to devise an algorithm for paying a given amount using smallest possible number of coins.

- E.g. if we want to pay Rs. 2.89 (289 paisa)

  Then the best solution is to give 10 coins:

  2 X 100 paisa = 200 paisa (2 coins)

  3 X 25 paisa = 75 paisa (3 coins)

  1 X 10 paisa = 10 paisa (1 coin)

  4 X 1 paisa = 4 paisa (4 coins)

  TOTAL = 289 paisa (10 coins)

Making Change (1) Problem

- This is example of Greedy Algorithm
- For this problem we are always getting a **Optimal Solution**; however with a different series of values, or if the supply of some of the coins is limited, the greedy algorithm may not work.

- The algorithm is "greedy" because at every step it chooses the largest coin it can, without worrying whether this will prove to be a sound decision in the long run.

General Characteristics of Greedy Algorithm

- To construct the solution of our problem, we have a set of candidates. *(Available coins)*
- As algorithm proceeds, we accumulate two other sets. One contains candidates that have already been considered and chosen, while the other contains candidates that have been considered and rejected.
- There is a function that checks whether a particular set of candidates provides a solution to our problem.
- A second function checks whether a set of candidates is feasible.
- The selection function, indicates at any time which of the remaining candidates, that have neither been chosen nor rejected, is the most promising.
- Finally, an objective function gives the value of a solution.

Greedy Algorithm

```
function greedy(C: set) : set
        {C is the set of candidates}
    s = Ø  {we construct the solution in the set S}
    while  c <> Ø  and not solution(s) do
        x = select(c)
        c = c \ {x}
    if feasible ( s U {x} ) then s = s U {x}
    if solution(s) then return s
    else return "No Solution"
```

Graphs: Minimum Spanning Trees

- Let G = <N,A> be a *connected, undirected graph.*

  where N is the set of nodes and A is the set of edges. Each edge has given length.

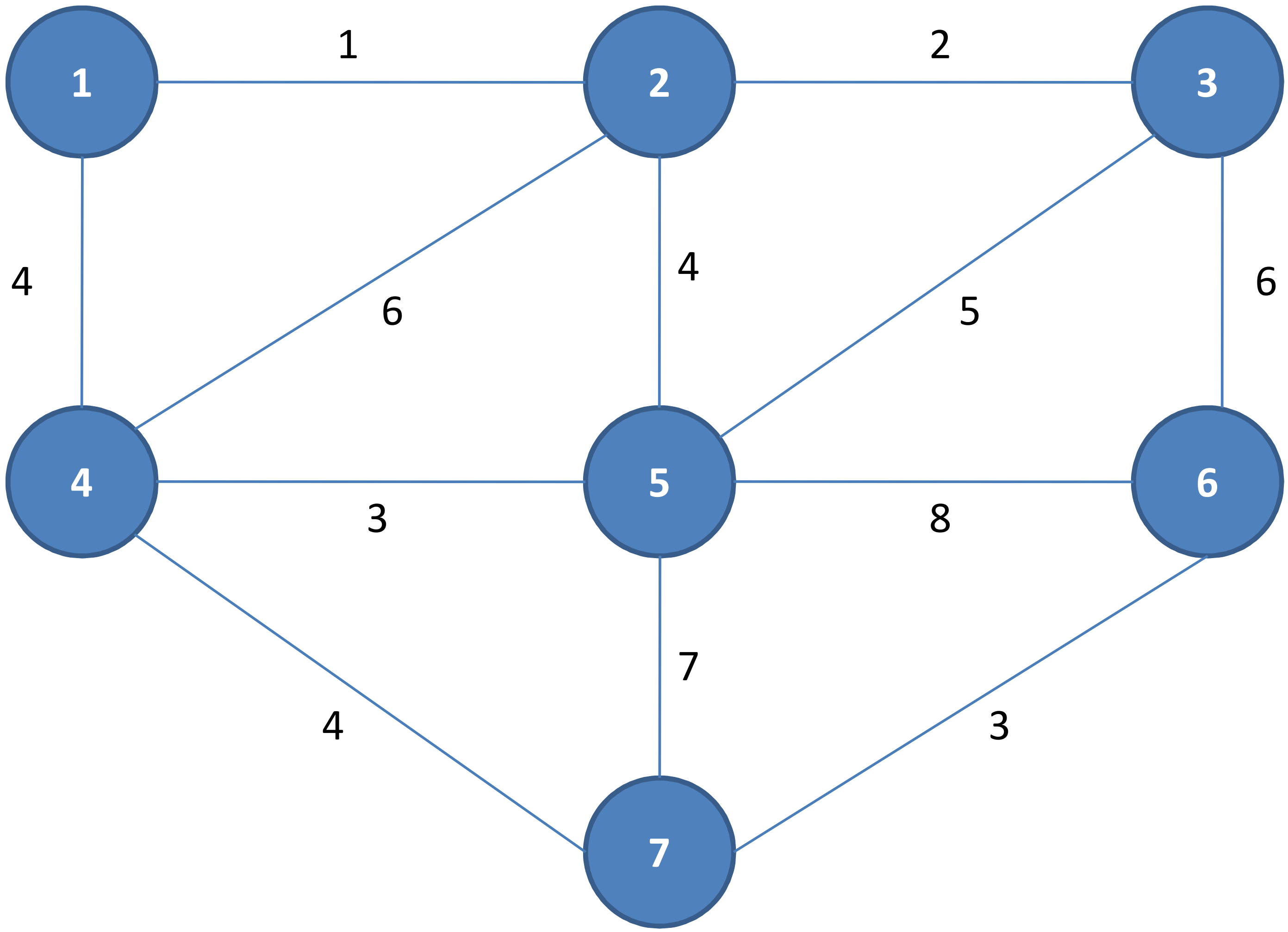
Problem: *The Problem is to find a subset T of the edges of G such that all the nodes remain connected, and the sum of the lengths of the edges in T is as small as possible.*

Note: A connected graph with n nodes must have at least n-1 edges, on other side, a graph with n nodes and more than n-1 edges contains at least one cycle.

Greedy Algorithm

- The candidates are the edges in G
- A set of edges in solution if it consists a spanning tree for nodes in N
- A set of edges is feasible if it does not include a cycle
- Objective is to minimize the total length

1. Kruskal's Algorithm (MST Problem)



Kruskal's Algorithm (MST Problem)

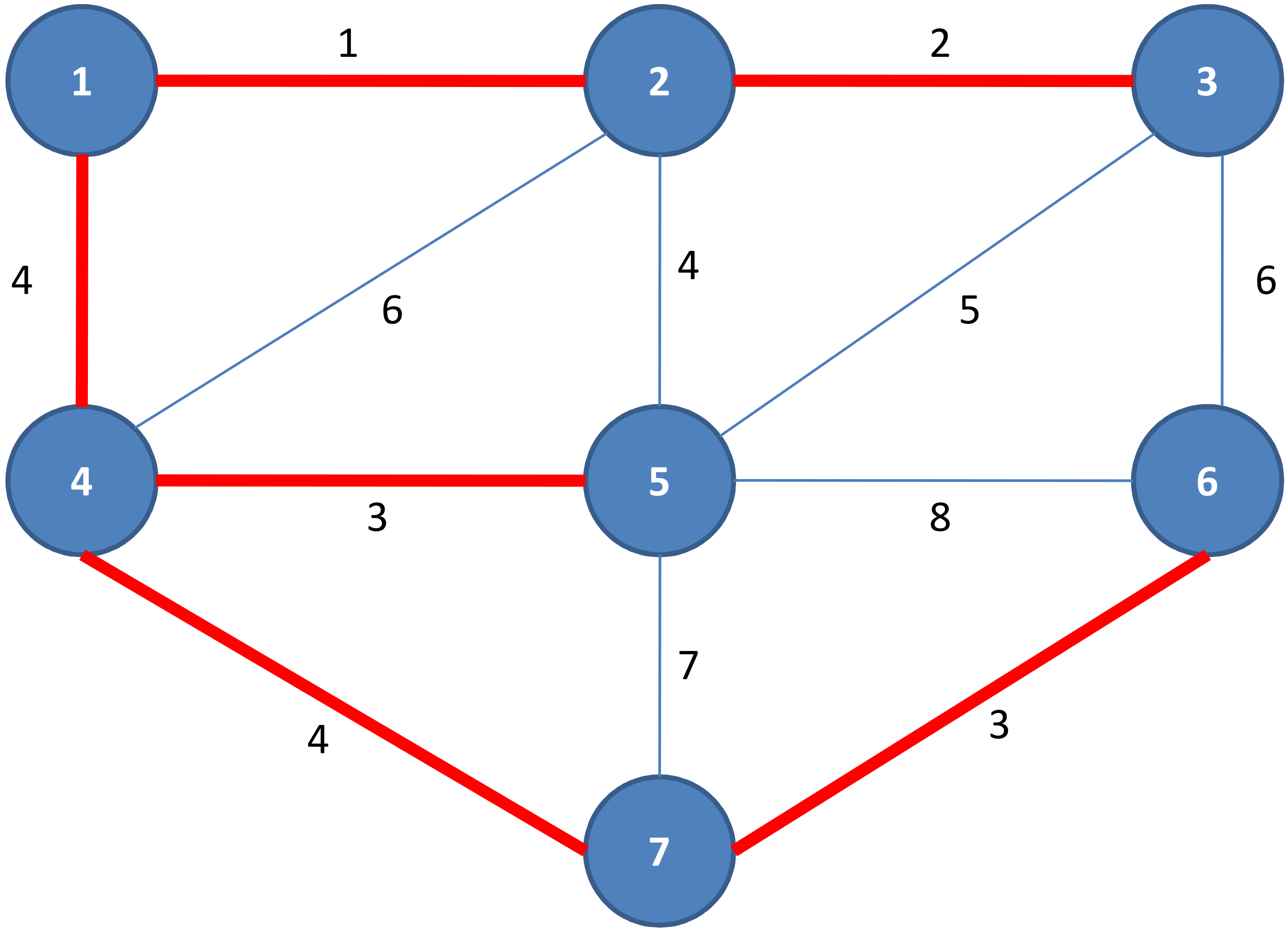
- Arrange all the edges of the graph in ***increasing order of their length***.
- So,

  {1,2}, {2,3}, {4,5}, {6,7}, {1,4}, {2,5}, {4,7}, {3,5}, {2,4}, {3,6}, {5,7} and {5,6}

Kruskal's Algorithm (MST Problem)

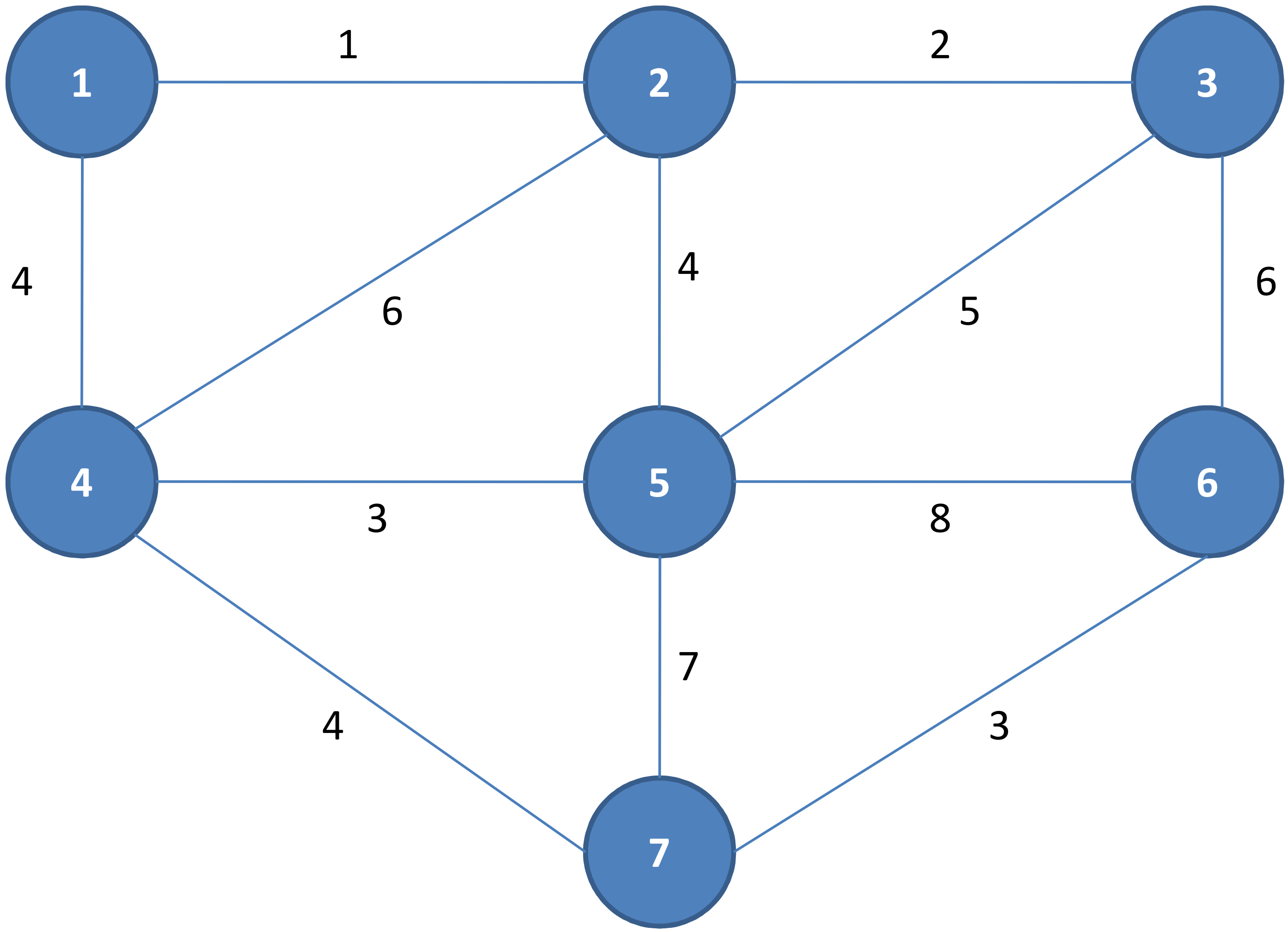
Step	Edge Considered	Connected Components
Initialization	--	{1} {2} {3} {4} {5} {6} {7}
1	**{1,2}**	{1,2} {3} {4} {5} {6} {7}
2	**{2,3}**	{1,2,3} {4} {5} {6} {7}
3	**{4,5}**	{1,2,3} {4,5} {6} {7}
4	**{6,7}**	{1,2,3} {4,5} {6,7}
5	**{1,4}**	{1,2,3,4,5} {6,7}
6	*{2,5}*	*Rejected*
7	**{4,7}**	{1,2,3,4,5,6,7}

Kruskal's Algorithm (MST Problem)



Total Length = 1+2+3+3+4+4= 17

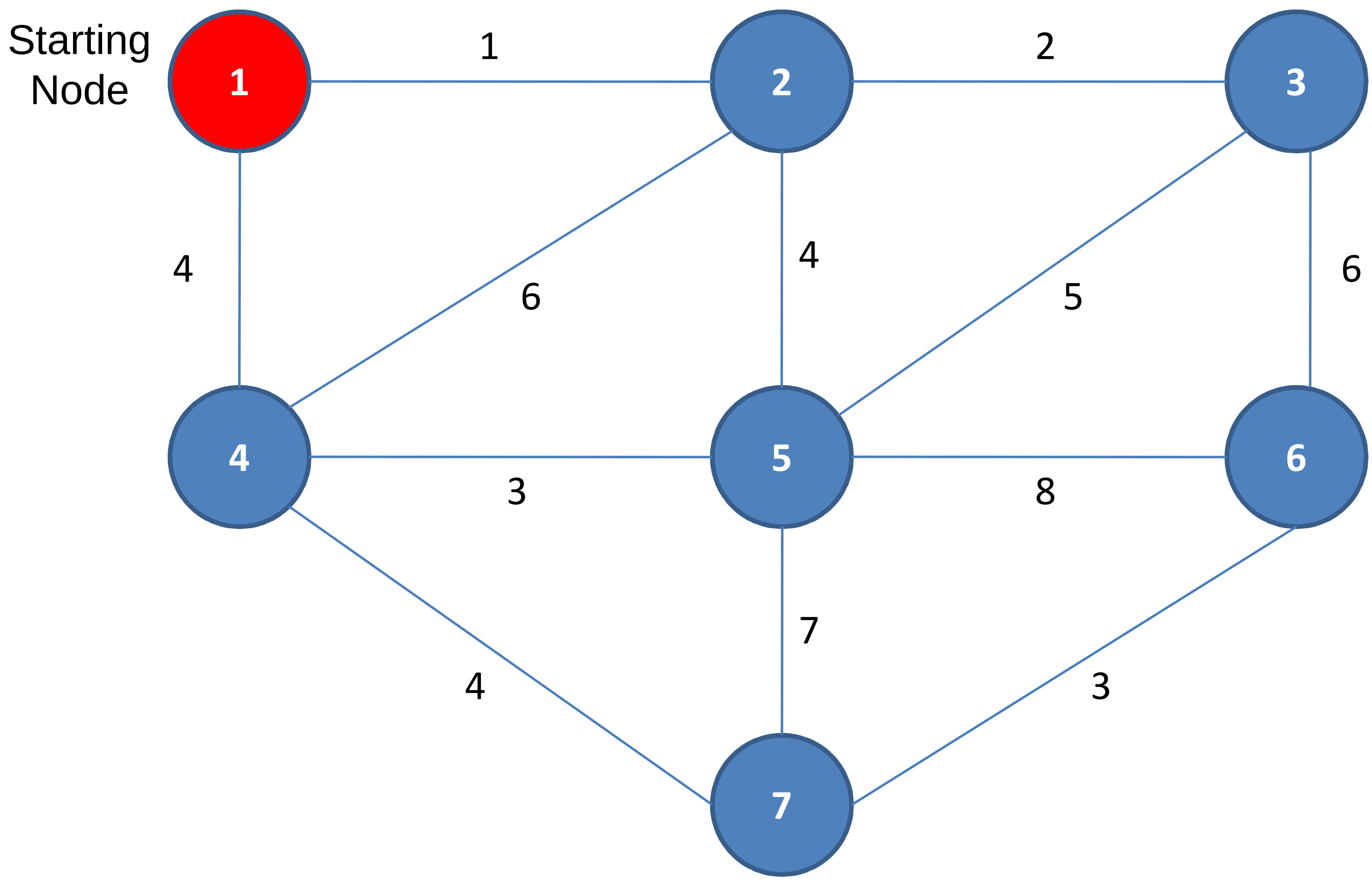
2. Prim's Algorithm (MST Problem)



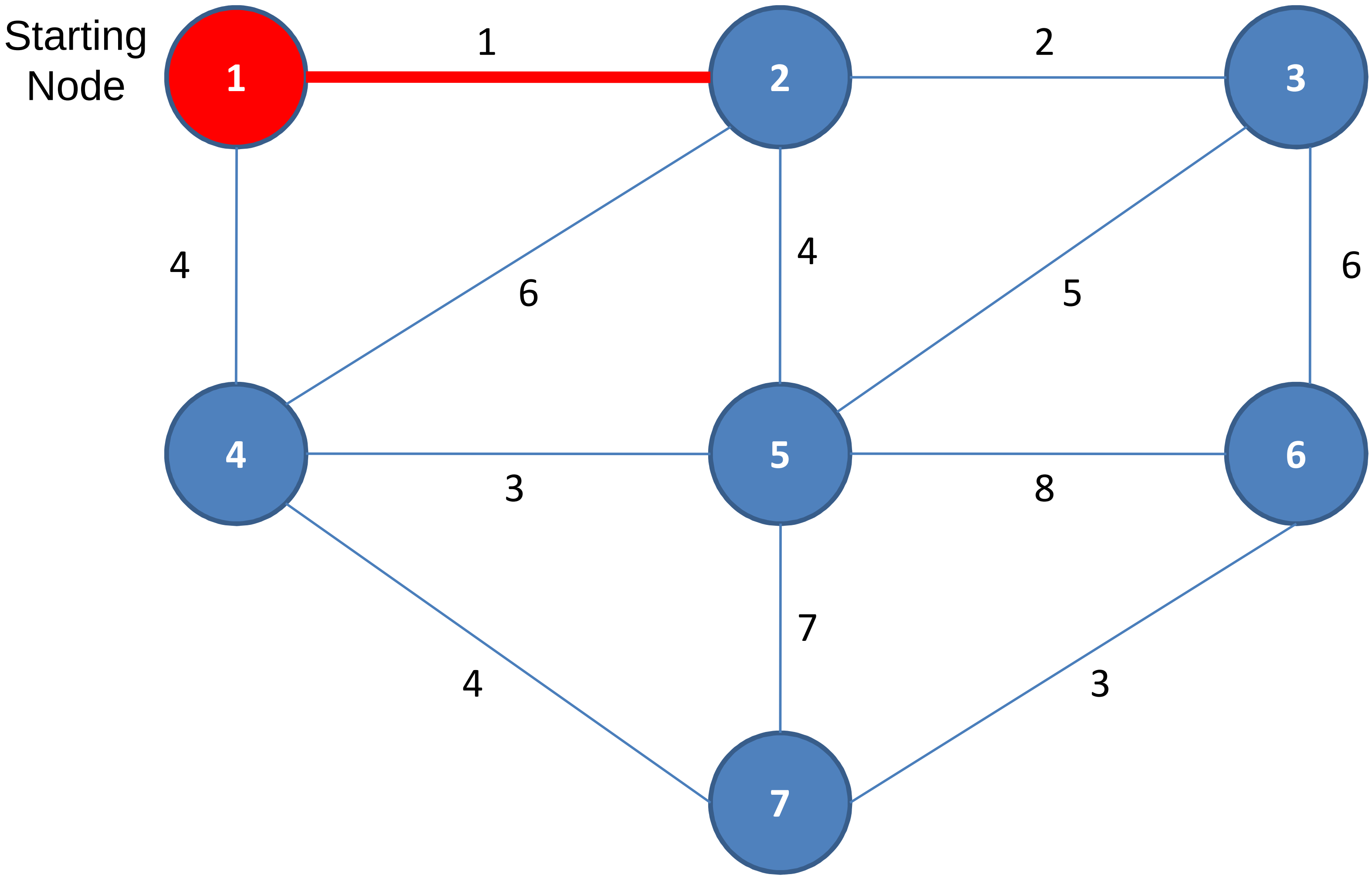
Prim's Algorithm (MST Problem)

- In this algorithm, the minimum spanning tree grows in a natural way, starting from an arbitrary root.
- At each stage we add a new branch to the tree already constructed.
- The algorithm stops when all the nodes have been reached.

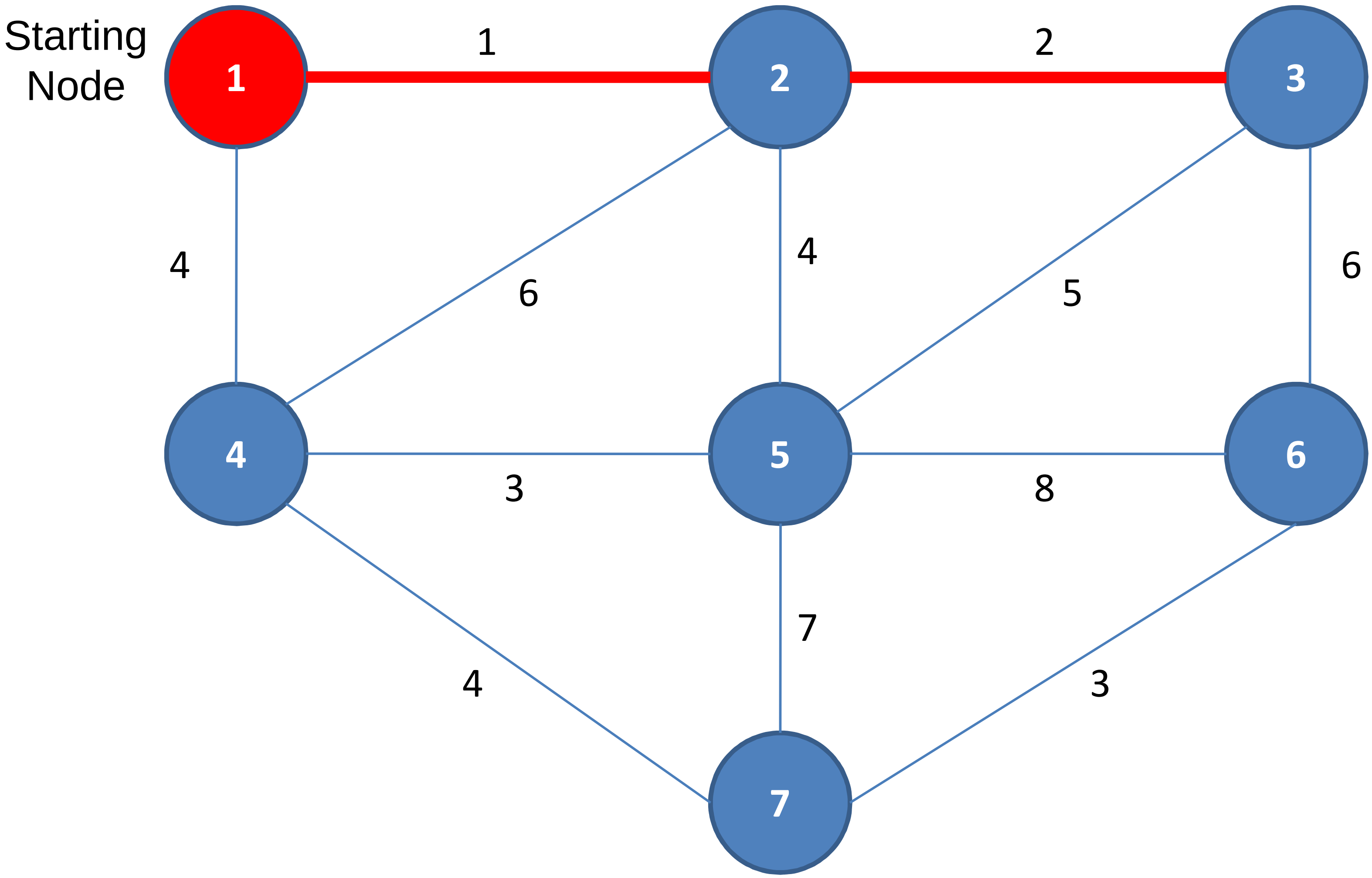
Prim's Algorithm (MST Problem)



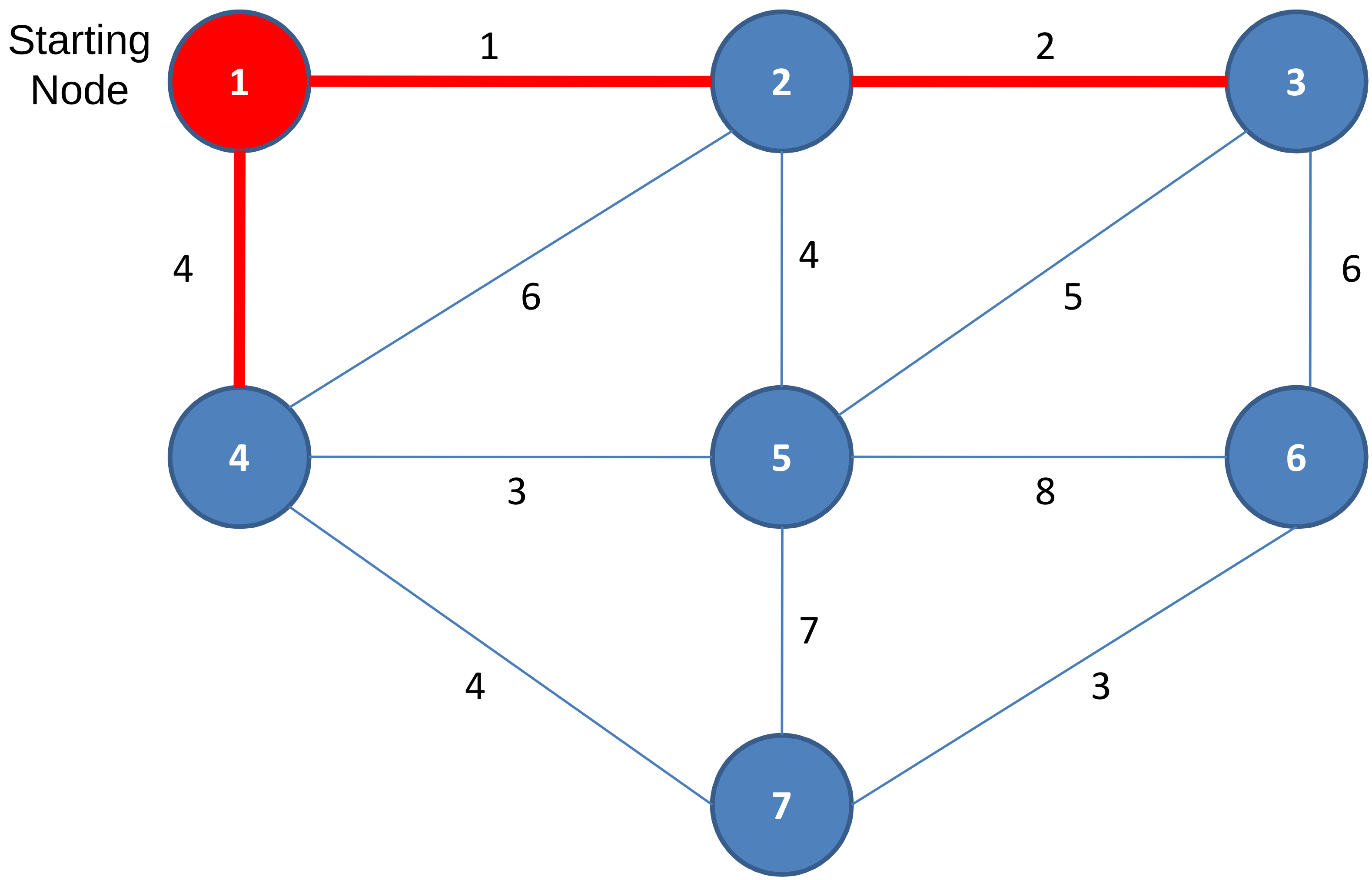
Prim's Algorithm (MST Problem)



Prim's Algorithm (MST Problem)



Prim's Algorithm (MST Problem)



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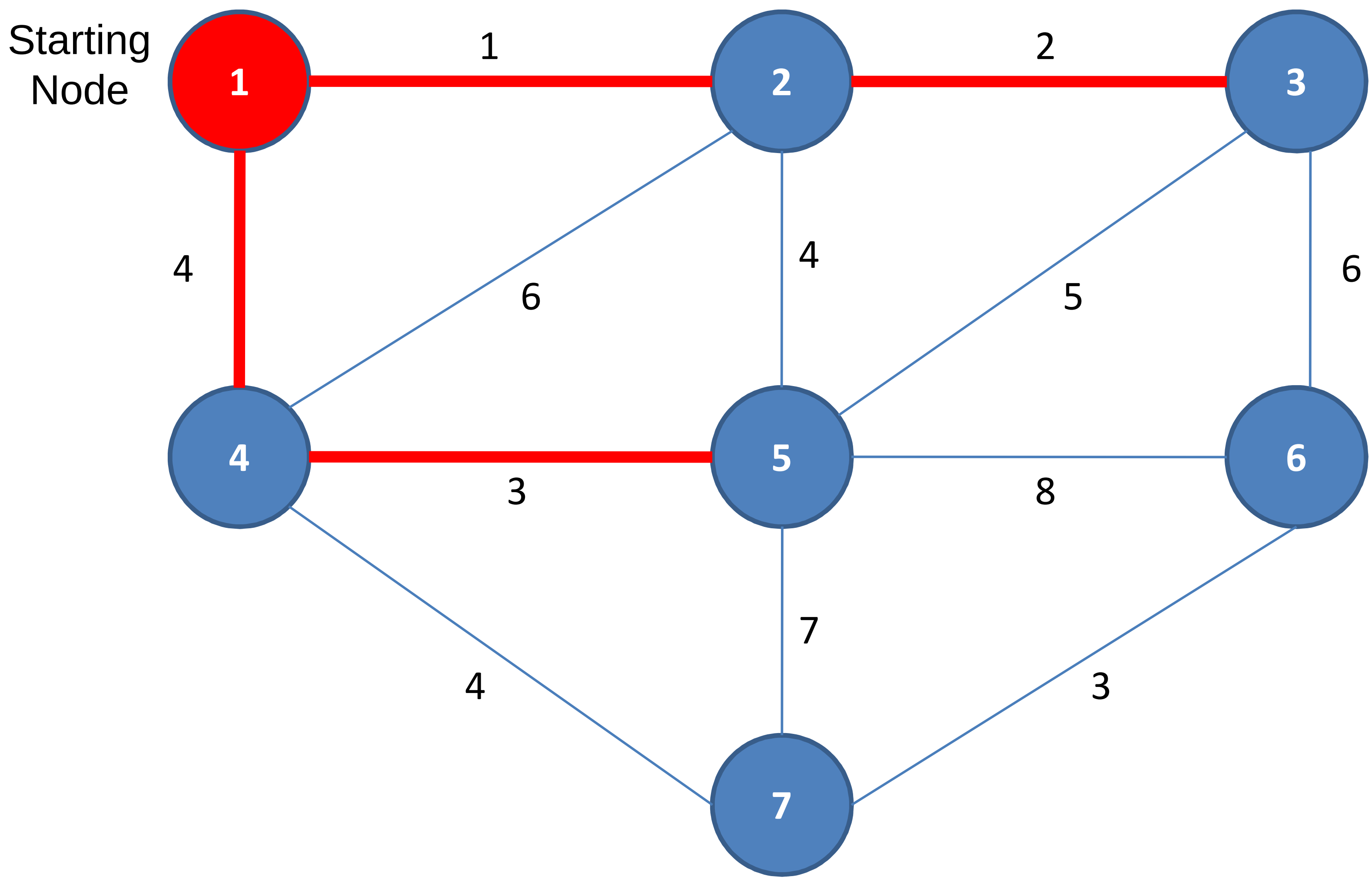

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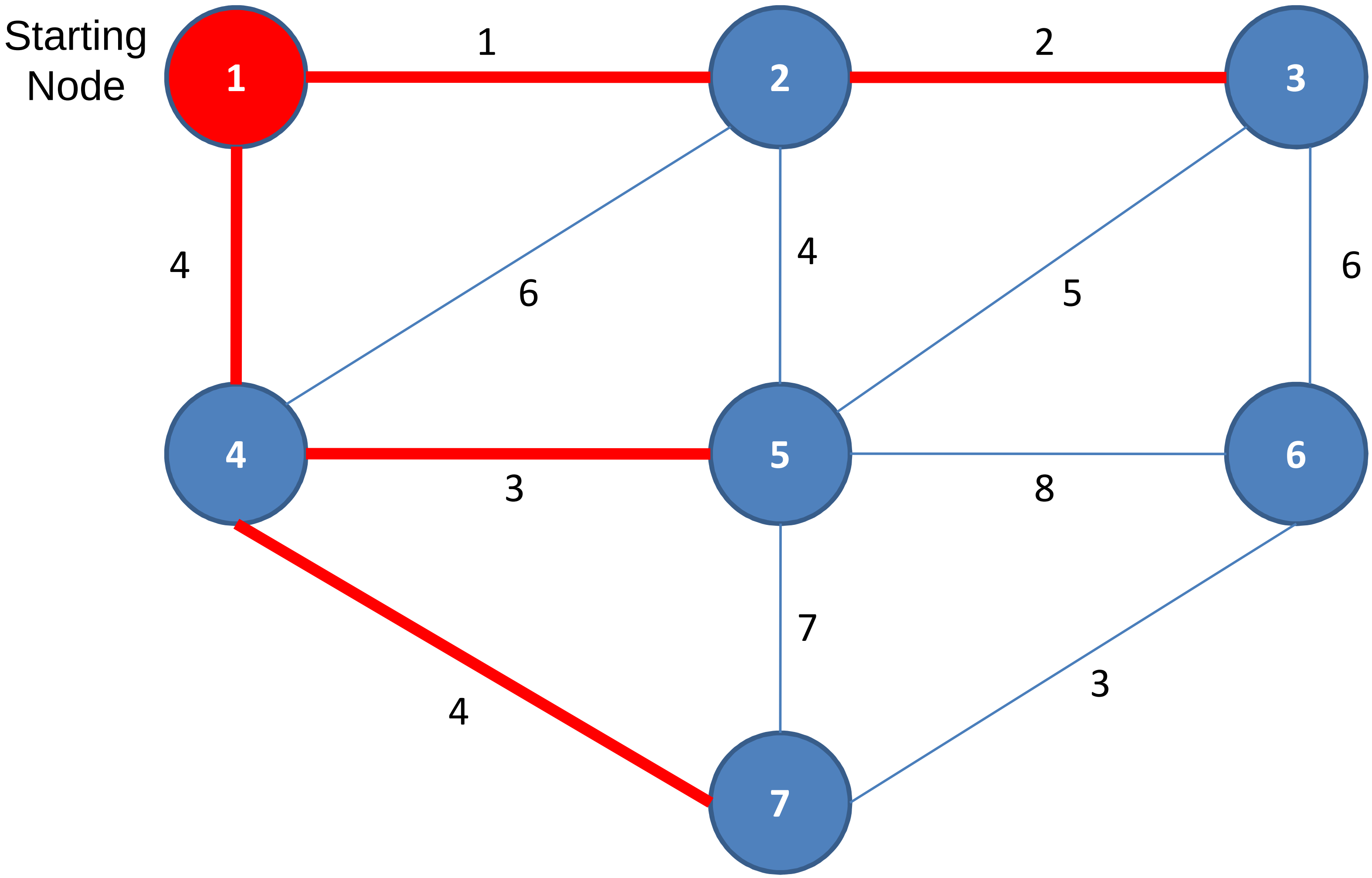

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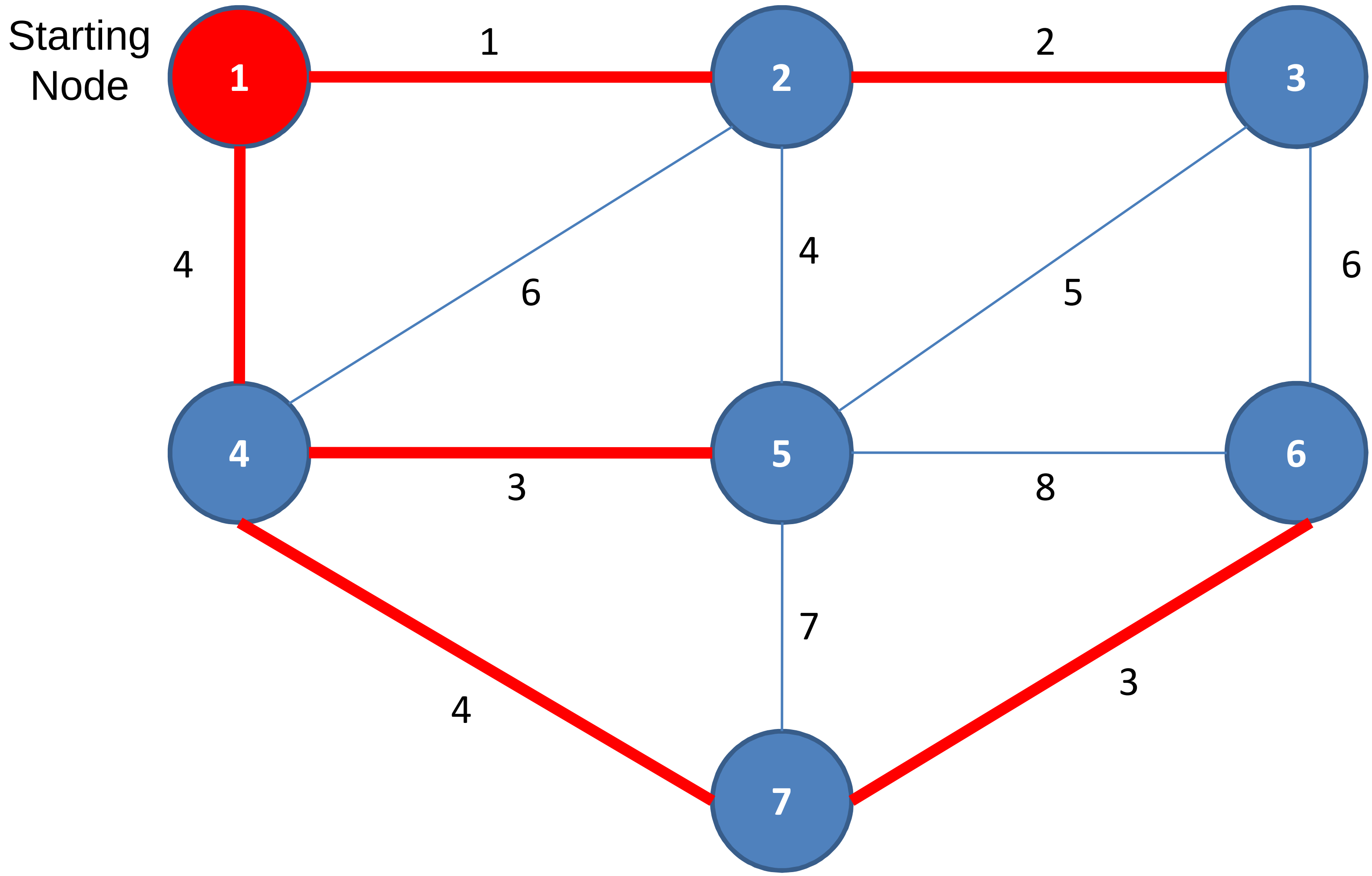

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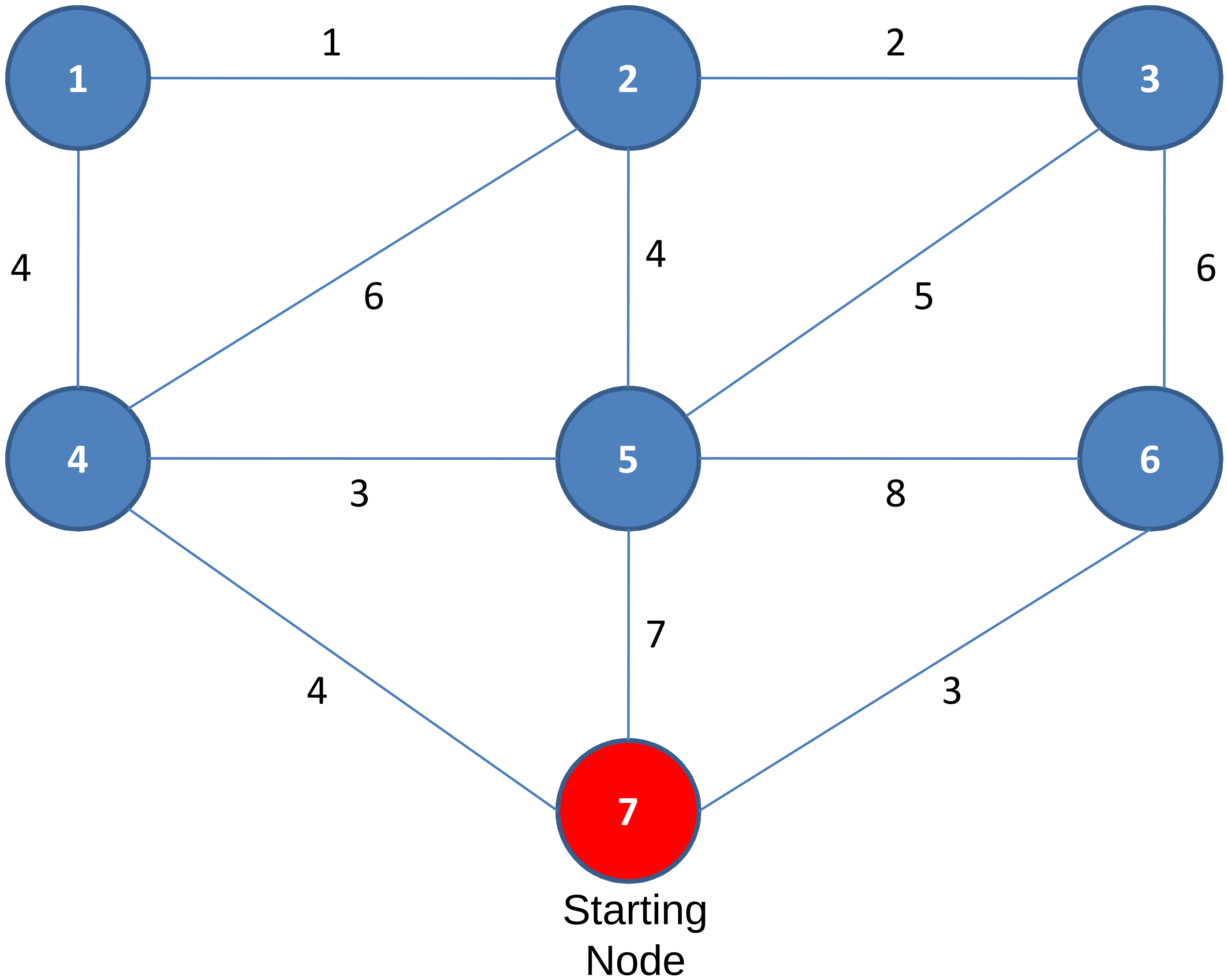
Prim's Algorithm (MST Problem)

Step	{U,V}	B
Initialization	--	{1}
1	{1,2}	{1,2}
2	{2,3}	{1,2,3}
3	{1,4}	{1,2,3,4}
4	{4,5}	{1,2,3,4,5}
5	{4,7}	{1,2,3,4,5,7}
6	{7,6}	{1,2,3,4,5,6,7}

Prim's Algorithm (MST Problem)



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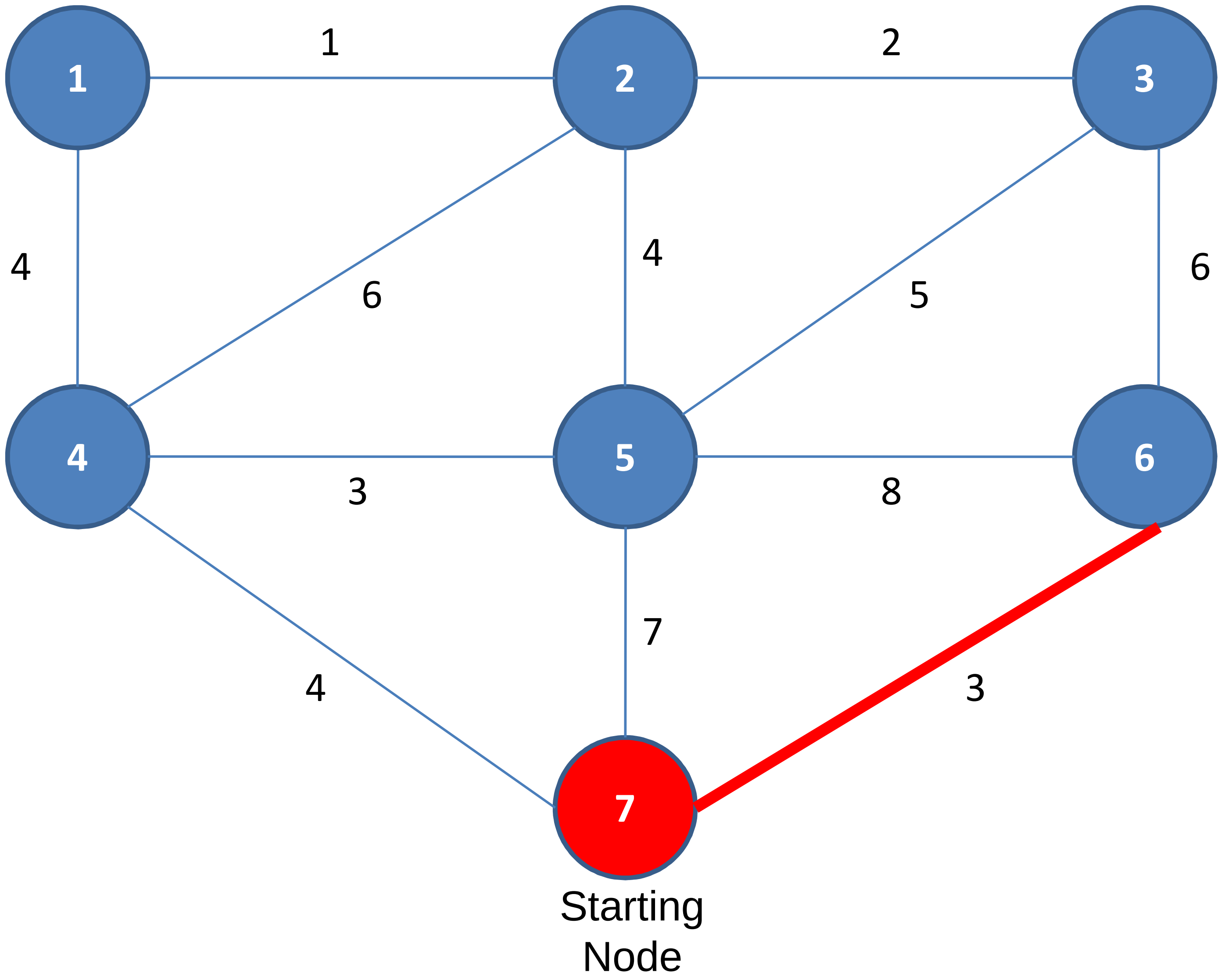

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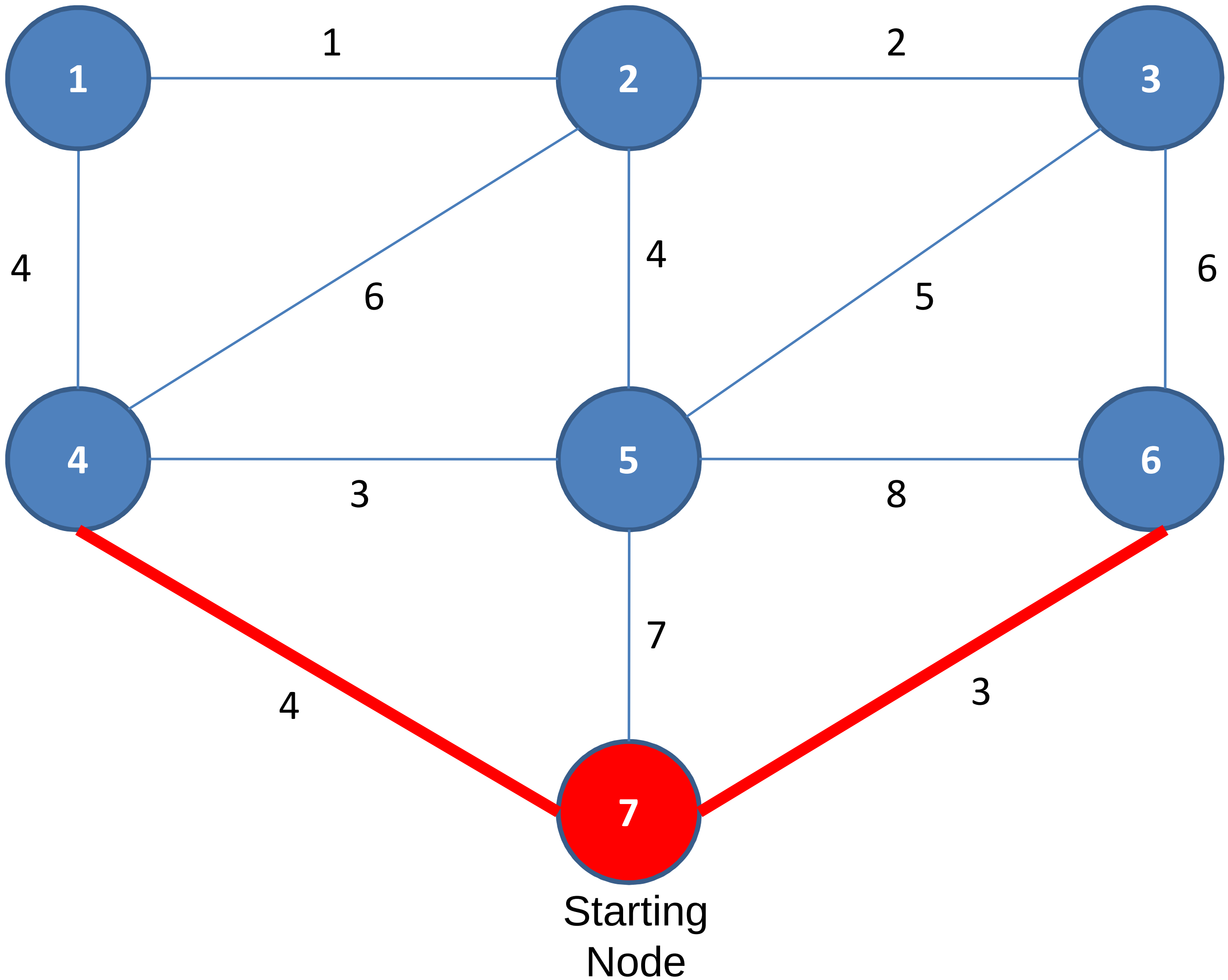

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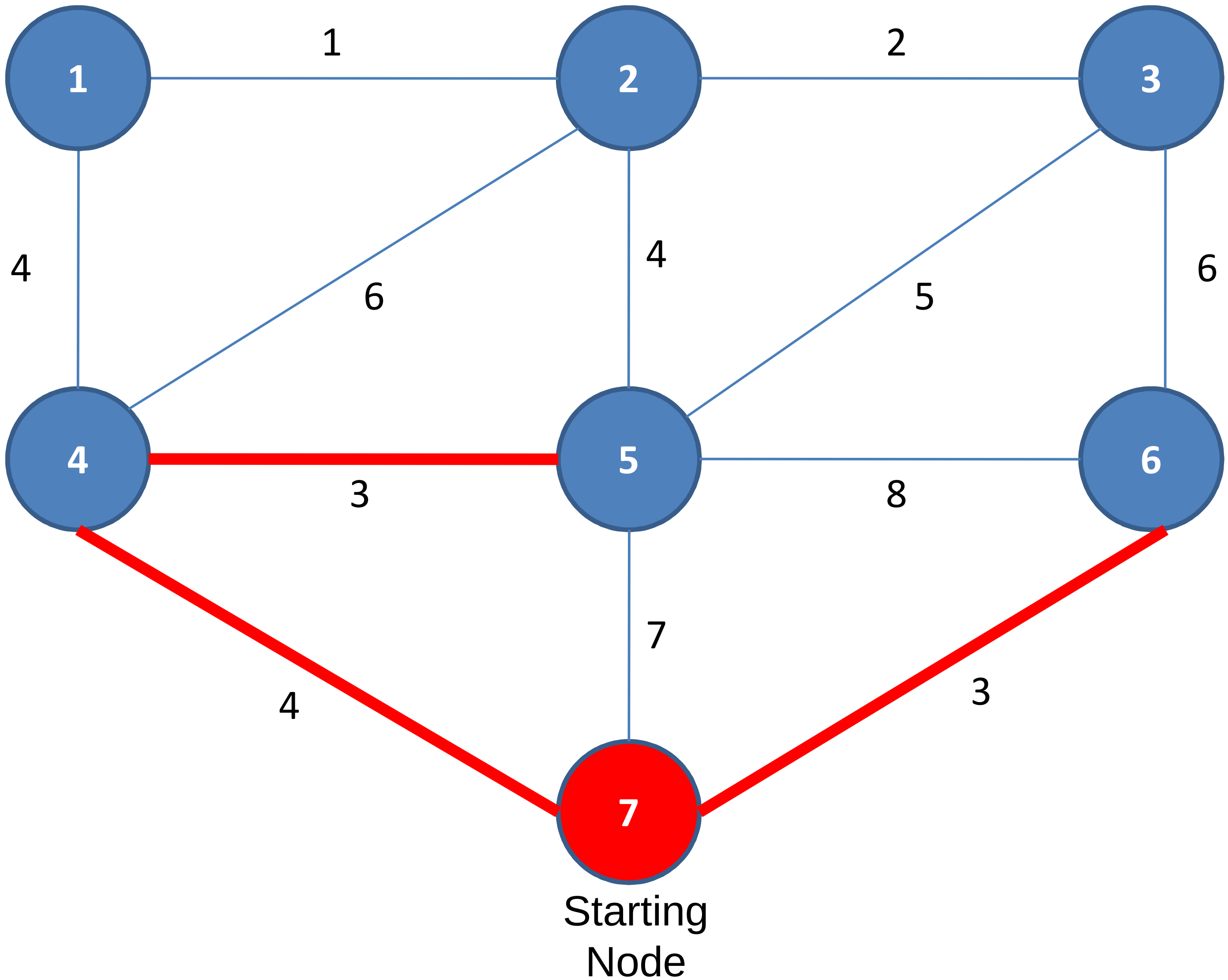

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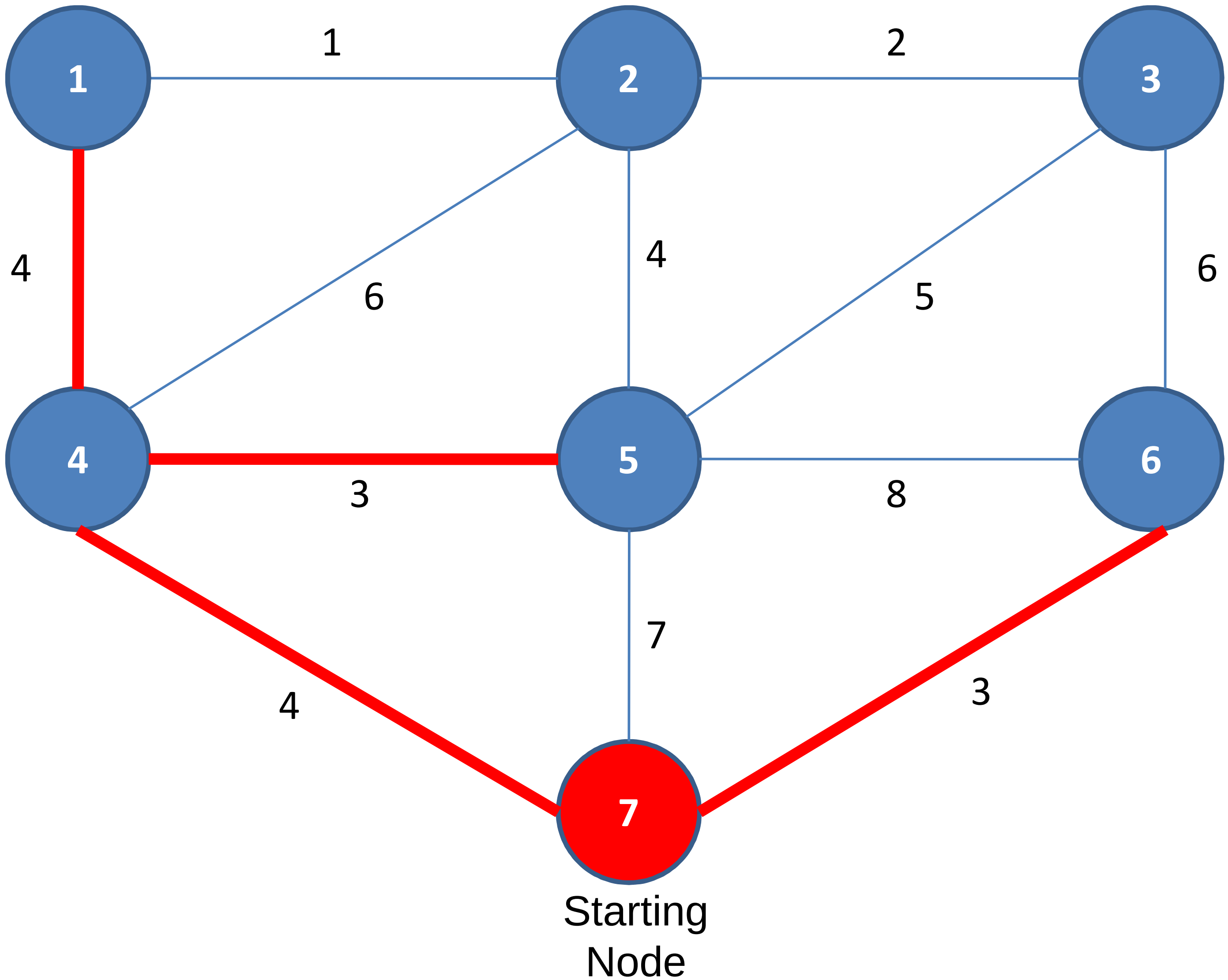

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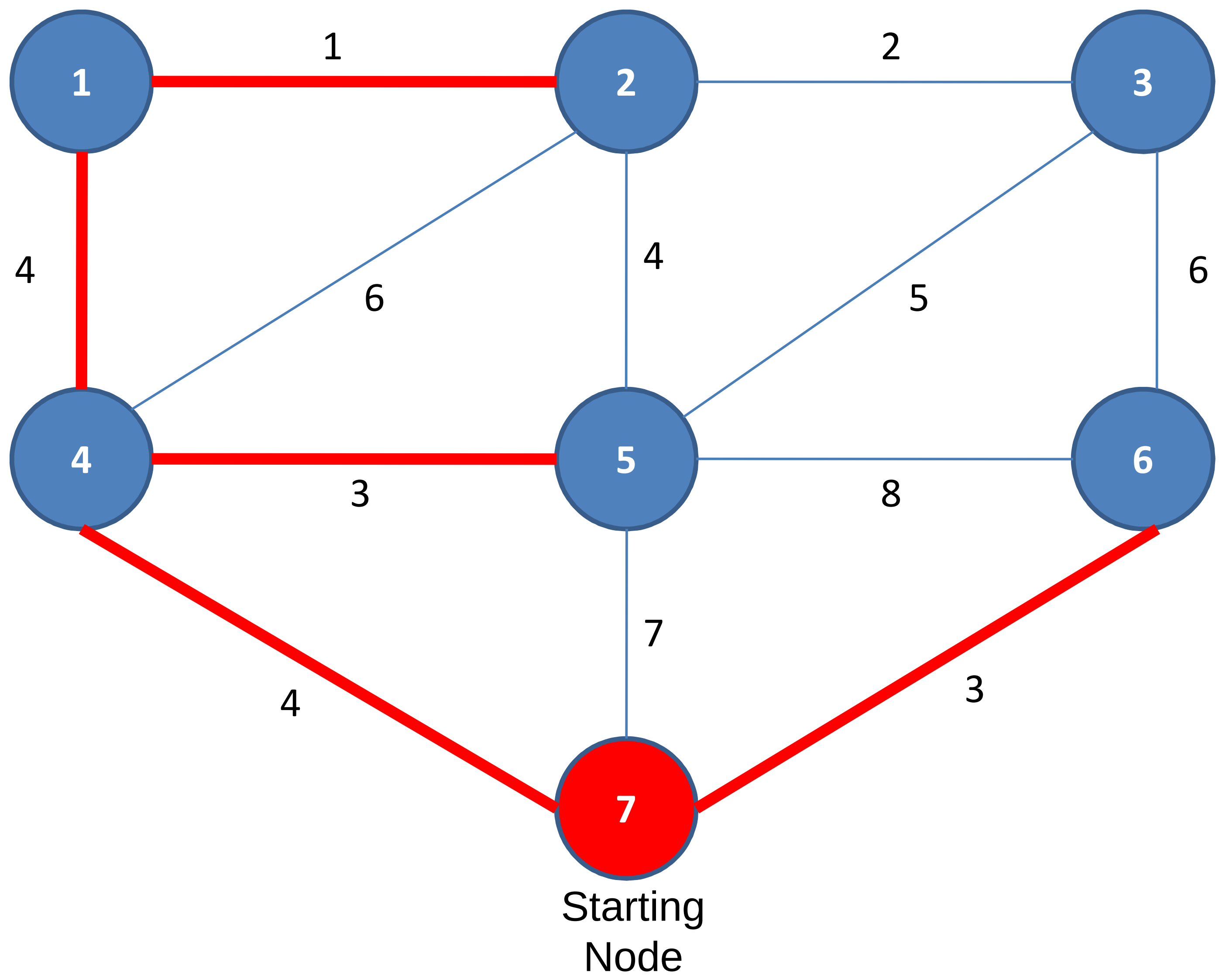

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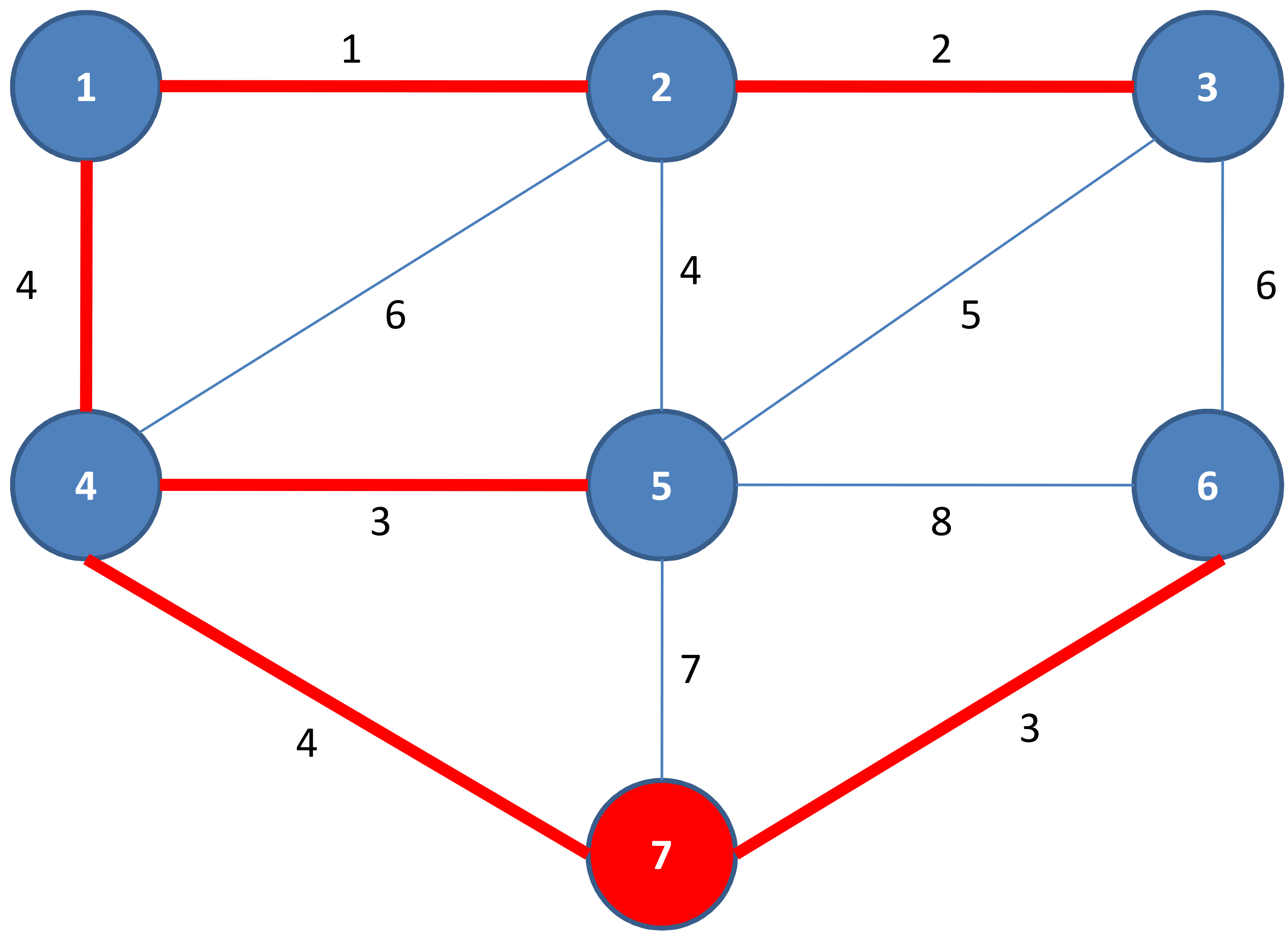


Total Length = 1+2+3+3+4+4= 17

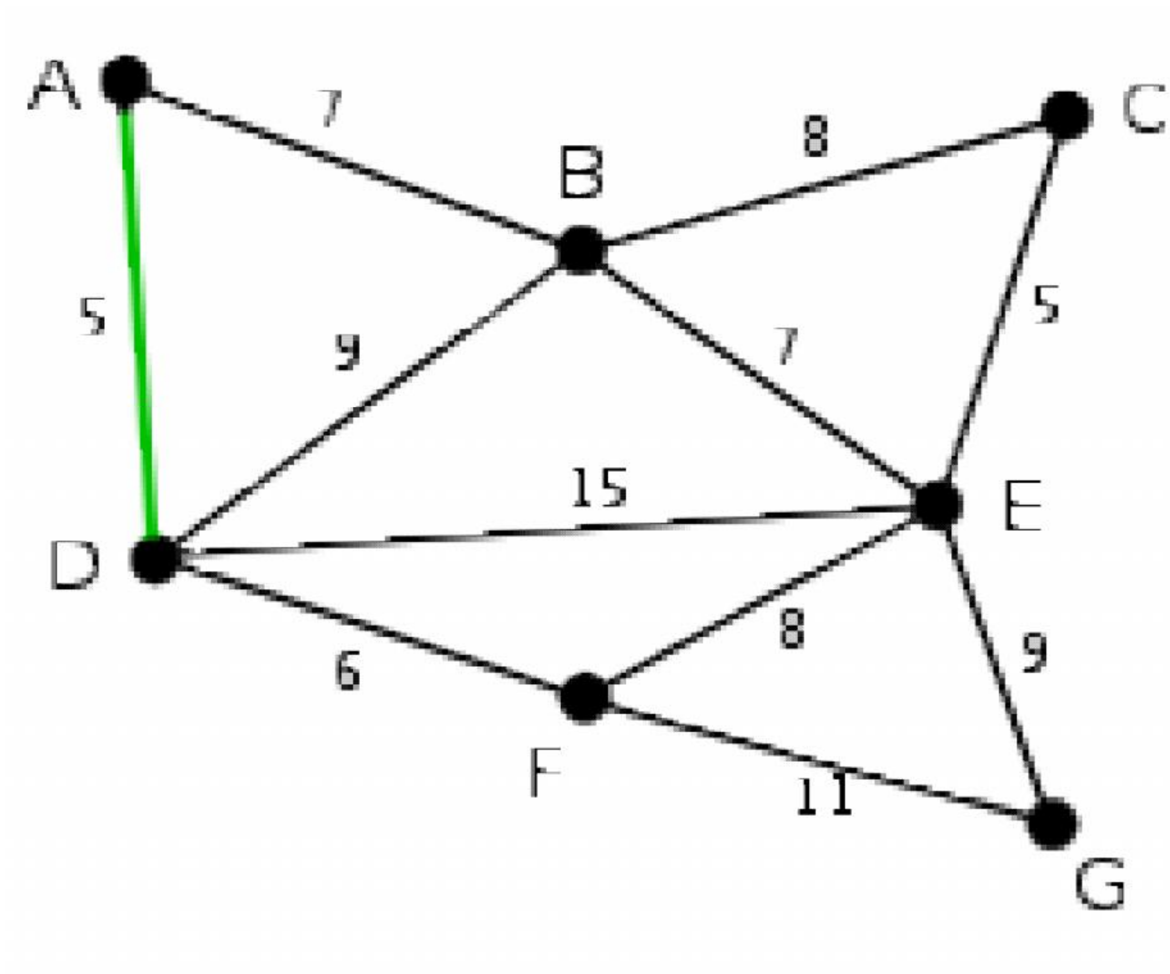
Prim's Algorithm (MST Problem)

Step	{U,V}	B
Initialization	--	{7}
1	{7,6}	{6,7}
2	{7,4}	{4,6,7}
3	{4,5}	{4,5,6,7}
4	{4,1}	{1,4,5,6,7}
5	{1,2}	{1,2,4,5,6,7}
6	{2,3}	{1,2,3,4,5,6,7}

Comparison of Kruskal’s & Prim’s Algorithm

- For a graph with **V** vertices **E** edges, Kruskal's algorithm runs in **O(E log V)** time and Prim's algorithm can run in **O(E + V log V)** amortized time.
- **Prim's algorithm is significantly faster** in the limit when you've got a really **dense graph** with many more edges than vertices. **Kruskal performs better** in typical situations (**sparse graphs**) because it uses simpler data structures.

Another Example (MST Problem)



Answer